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Slew Calibration for Spin-Stabilized Satellites

Luc Fraiture*
European Space Operations Centre, D64293 Darmstadt, Germany

The slewing of the rotation axis of a spin-stabilized satellite along a rhumb line by means of thrust pulses is addressed. It is shown that one can calibrate the mean thrust level and the mean pulse centroid delay independently from attitude knowledge before and beyond the slew. To achieve this one has to consider two successive slews or a slew broken down into two legs with different rhumb angles. The basic condition is that the thruster behavior is approximately equal in both legs. The required data are the solar aspect angles each time just before and just beyond the reorientation maneuvers. For the special case where the breakdown is made in two equal legs, called doglegs, it has been investigated which path increase has to be expected and to which extent the probability increases to end with a larger off-target error than with a single direct slew. Also the extrapolation of the initial attitude by using the planned maneuver data with corrrected calibration has been inspected by means of covariance analyses. In all of these aspects, the results of analyses are favorable. Finally, the calibration technique has been applied in three very large maneuvers on three similar spacecraft and the corresponding operational results are discussed.

Nomenclature

k	=	pulse strength calibration factor
$\boldsymbol{N}$	=	spin axis direction unit vector
$\boldsymbol{S}$	=	sun direction unit vector
Δ_{att}	=	angular difference between attitude estimates, deg
δ	=	declination in a sun coordinate system, rad
ϵ	=	rhumb angle correction term, rad
η	=	selected rhumb angle offset, rad
θ	=	sun angle, rad
λ	=	right ascension in a sun coordinate system, rad
ρ	=	rhumb angle of a maneuver on the Mercator projection, rad
σ_i	=	standard deviation of parameter i
ϕ	=	maneuver pathlength, rad
$\{\ \}_{0/e}$	=	parameter for initial/final attitude of a maneuver

Introduction

THE reorientation of the rotation axis of a spin-stabilized satellite is performed by operating thrusters in pulsed mode. The relevant thruster must have a thrust component parallel to the spin axis. The direction of the slew depends on a phase angle, called rhumb angle, with respect to an inertial reference. Therefore, the same thruster(s) can be employed for any reorientation direction, in contrast to slewing techniques on three-axis stabilized satellites, where not the angular momentum vector is moved, but the satellite reference frame is rotated as a whole. Rhumb line maneuvers for spinning satellites are well known, and their succinct description can be found in the contribution of Williams in Ref. 1 (pp. 651–654).

The thrust engines utilized for rhumb line maneuvers on spinning spacecraft are usually slightly stronger than the thrusters on a three-axis stabilized craft, but they also rely on cold gas, hydrazine, or bipropellants either working in pressure-regulated or blowdown mode. Sizable attitude reorientation maneuvers of spinning satellites are critical events, which one normally monitors in real time. Aspects playing a central role at these occasions are as follows: 1) ability to determine quickly the attitude to cope with maneuvers that deviate from their original path and do not end with the expected attitude sensing coverage required for a complete attitude assessment, 2) avoidance of station contact loss by selecting slew paths that maintain radio frequency link capability with the spacecraft during the maneuver, and 3) application of past thruster calibration and extraction of new calibration after the event. Especially the first aspect has been a point of concern for very large slew demands where the present procedure, subdividing a slew into two differently oriented legs, provides a method to deal with contingent maneuver end attitudes. However, both other aspects get important benefits from the algorithm described in this paper. In particular, one is able to extract calibration data from the maneuver without intervention of the actual attitude, by using only a succession of measured solar aspect angles. In this way one eliminates the influence of the attitude estimation accuracy on thruster calibration. Thus, this is also useful if one only disposes of short maneuvers for calibration purposes. As the method we propose consists in breaking down a maneuver in two legs, parameters of the subdivision of the path can be employed to select a route along attitudes that maintain the radio frequency link in a given mission phase. A further asset of the calibration method results from that the algorithm does not imply any simplifying assumption or approximation. It is only subject to the hypothesis that the applicable calibration is equal in the two maneuver legs one is using in the calibration procedure.

One may have gained the impression that the calibration method we propose requires a specifically planned subdivision of a maneuver into two parts to be applicable. Such a planning is not necessary as long as one can assume that two successive maneuvers with the same thruster(s) are subject to the same calibration, that is, performances are repeatable. Anyway calibration is only meaningful in the context of repeatability. This is usually not a problem for cold gas and biliquid thrusters with pressure-regulated fuel supply modes or very small blowdown of the tank pressure. When working with such engines one can accept that two maneuver legs have occurred at very different times with different path lengths, and their combination for the purpose of calibration poses no problem. Repeatability is a problem for catalytic hydrazine decomposition thrusters. The latter represent a large class of thrusters used in practice. The catalyst inside these thrusters is heating during the first pulses of a train, and this heating is a function of the thruster on/off duty cycle, the fuel feed pressure, the initial thrust chamber temperature, and the catalyst status. During this heating, the performance of the thruster changes notably. The calibration figures one can extract after the maneuver are mean values also comprising the initial variable performances. The corresponding calibration of catalytic hydrazine decomposition thrusters is thus only applicable to a maneuver that is almost identical with respect to number of pulses, on/off duty cycle, fuel feed pressure, and initial thruster temperature. If, in addition, the hydrazine feed pressure is subject to strong blowdown (the early maneuvers on systems with blowdown ratios up to 4:1), similar conditions can only approximately be guaranteed even in successive maneuvers, that is, two maneuvers not separated by any other maneuver based on the same reaction control system. Part

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*Head, Spacecraft Attitude Section (retired); current address Lucasweg 6, D64287 Darmstadt, Germany.

of the present contribution aims at the description of a procedure to create repeatability conditions by subdividing maneuvers in two parts of equal length, with different rhumb angles but with similar on/off duty cycles. We call them dogleg maneuvers.

This paper is organized as follows. In the next section the calibration principle is introduced against the general background of rhumb line maneuver paths in a Mercator projection where the sun direction is at 90-deg latitude. In a further section we discuss performance for the special case of dogleg maneuvers. We will look first at the path length increase of a dogleg with respect to a single leg maneuver, that is, the price of a dogleg in terms of fuel. The next performance aspect is the off target error change brought about by a dogleg. We mean that if the direct single slew ends off the intended attitude target, by how much do we worsen the situation if we employ a dogleg. Finally, we investigate the accuracy of the end attitude estimate obtained by the present calibration method using the initial attitude and the succession of solar aspect angles in the particular case of dogleg maneuvers. In a last section we briefly describe the three large dogleg maneuvers which have been implemented so far at the European Space Operations Centre (ESOC), the control center of the ESA.

Calibration Principle

Note that the sun angle θ of the spacecraft is defined by $(\boldsymbol{N} \cdot \boldsymbol{S}) = \cos\theta$. Alternative names in common use for θ are solar aspect angle and sun colatitude. Rhumb lines for spinning satellites are normally defined on a sphere where $\boldsymbol{S}$ corresponds to the North pole and where the rhumb angle is defined with respect to the meridians issuing from the polar axis. The corresponding reference system is called a sun coordinate system. On a plane Mercator projection of a sun coordinate system, where $\theta = 90$ deg corresponds to the equator, the rhumb lines are straight lines making a fixed angle, the rhumb angle ρ, with the parallel meridians. The λ and δ of the inertial spin axis orientation correspond to abscissas and ordinates in the Mercator projection, respectively. We note that $\delta = \pi/2 - \theta$. What we miss, to map any attitude on the Mercator projection, is an origin, that is, the meridian corresponding to $\lambda = 0$. Usually this is chosen to be the meridian on which we find the initial attitude or the begin point of the rhumb line.

The projection onto a meridian of a differential path change $\mathrm{d}\phi$ along a rhumb line is equal to $\mathrm{d}\phi \cos\rho = \mathrm{d}\delta$. The same change projected on a small circle at latitude δ on a unit sphere yields $\mathrm{d}\phi \sin\rho = \mathrm{d}\lambda \cos\delta$. By integrating these equations, we find the well-known relations

$$\ln\left\{\frac{\tan[0.5(\pi/2 + \delta_e)]}{\tan[0.5(\pi/2 + \delta_0)]}\right\} = (\lambda_e - \lambda_0)\,\mathrm{cotan}\,\rho \tag{1}$$

$$\phi\cos\rho = \delta_e - \delta_0 \tag{2}$$

We do not need to go into the details of the translation in general use of the rhumb line parameters to thruster related parameters. We mean the transformation of path length into number of pulses and rhumb angle into pulse start phase delay. These two transformations are normally subject to errors that can, to a certain extent, be calibrated. The mean pulse strength may be in error by a factor k, so that in the end not ϕ but a path $k\phi$ is accomplished. Also the mean pulse delay is subject to the sum of many error contributions, which make a maneuver progress along a rhumb line with rhumb angle $\rho + \epsilon$ instead of ρ. If one can assume that k and ϵ are the same in two given maneuvers, one with $(\phi_1, \rho_1, \delta_{10}, \delta_{1e})$ and the other with $(\phi_2, \rho_2, \delta_{20}, \delta_{2e})$, Eq. (2) can be rewritten twice with the two unknown calibration parameters k and ϵ, namely,

$$k\phi_1\cos(\rho_1 + \epsilon) = \delta_{1e} - \delta_{10} = \theta_{10} - \theta_{1e} \tag{3}$$

$$k\phi_2\cos(\rho_2 + \epsilon) = \delta_{2e} - \delta_{20} = \theta_{20} - \theta_{2e} \tag{4}$$

In these equations ϕ_1, ρ_1, ϕ_2, and ρ_2 are known from the maneuver planning and command implementation and θ_{10}, θ_{1e}, θ_{20}, and θ_{2e} are known as solar aspect angle measurements made at the start and the end of each leg. Two unknowns are left, namely, k and ϵ. Their resolution yields

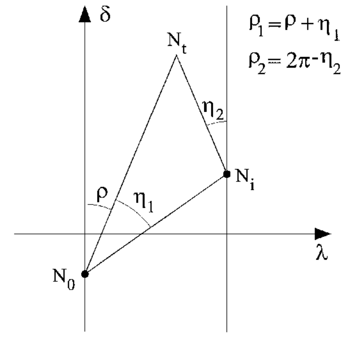

Fig. 1 Direct maneuver compared to a dogleg in a Mercator projection; note that the actual path length equality $\phi(N_0, N_i) = \phi(N_i, N_t)$ is distorted by the projection.

$$\tan\epsilon = \frac{(\theta_{20} - \theta_{2e})\phi_1\cos\rho_1 - (\theta_{10} - \theta_{1e})\phi_2\cos\rho_2}{(\theta_{20} - \theta_{2e})\phi_1\sin\rho_1 - (\theta_{10} - \theta_{1e})\phi_2\sin\rho_2} \tag{5}$$

and by substituting ϵ in one of the original equations, one finds

$$k = \frac{(\theta_{i0} - \theta_{ie})}{\phi_i\cos(\rho_i + \epsilon)} \qquad (i = 1 \quad \text{or} \quad 2) \tag{6}$$

provided $\rho_1 \neq \rho_2$. This result demonstrates that the calibration is independent from λ, that is, independent from a rotation of the rhumb lines around the sun direction.

Performance of Dogleg Maneuvers

Dogleg maneuvers are two-legged slews in which each leg has (approximately) the same designed path length. The situation is shown in Fig. 1, where N_0, N_i, and N_t are the initial, intermediate, and target attitude points on a Mercator projection, respectively. Conventionally, the direct maneuver with rhumb angle ρ slews the spin axis from N_0 to N_t, whereas with the dogleg, the first slew goes from N_0 to N_i on a path with rhumb angle $\rho_1 = \rho + \eta_1$, and the second goes from N_i to N_t with the rhumb angle $\rho_2 = 2\pi - \eta_2$. That the path length of the legs is equal can formally be written as $\phi(N_0, N_i) = \phi(N_i, N_t)$. For the free parameter in the dogleg design, we have selected the value of η_1, the offset angle of the first leg. The value of η_2 can be found by means of a little search algorithm. One can, for instance, fix the path length of the first leg initially to $\phi_1 = 0.45\,\phi(N_0, N_t)$, compute the resulting N_i, and from there find the corresponding $\phi_2 = \phi(N_i, N_t)$. The latter will normally be larger than ϕ_1. We now increment slightly ϕ_1 and repeat the procedure until the point where $\phi_1 > \phi_2$. This event indicates that we have passed the point of equality of path lengths. This procedure can further be refined to the required degree of precision, yielding the value of η_2 and the path length $\phi_{1,2}$ common to both legs, corresponding to the original direct slew parameters ρ and ϕ and the design parameter η_1.

Relative Path Length Increase of Dogleg Maneuvers

To assess how large the increase of the sum of the legs is with respect to a direct slew, we have compiled Table 1. For this purpose we have selected a path length of a direct maneuver equal to 40 deg and a start declination on the Mercator projection equal to -20 deg. The table indicates that offset angles up to $\eta_1 = \pm 10$ deg correspond to a negligible path increase. Up to ± 30 deg, the relative increase is below 20%, and this is usually acceptable in return for the benefits doglegs provide in an operational context.

Off-Target Errors of Dogleg Maneuvers Compared to Direct Slews

One of the purposes of dogleg maneuvers is to perform slews with inaccurate a priori calibration. Thus, with a direct slew we also expect to miss the target by a certain angle, which can statistically be predicted by means of an error analysis. The same applies to dogleg

Table 1 Ratio of total dogleg path lengths with respect to the direct maneuver path length of 40 deg and a start declination $\delta_0 = -20$ deg

η_1, deg	ρ = 0 deg	ρ = 30 deg	ρ = 60 deg	ρ = 90 deg	ρ = 120 deg
−60	2.00	2.00	2.00	2.09	2.02
−40	1.31	1.31	1.33	1.36	1.35
−30	1.15	1.16	1.15	1.19	1.15
−20	1.06	1.07	1.07	1.09	1.09
−10	1.02	1.02	1.02	1.03	1.03
+10	1.02	1.02	1.01	1.00	1.00
+20	1.06	1.06	1.05	1.04	1.02
+30	1.15	1.16	1.15	1.11	1.15
+40	1.31	1.31	1.28	1.22	1.17
+60	2.00	2.00	2.03	1.71	1.51

Table 2 Standard deviations in deg of the off-target error of direct and dogleg maneuvers with η_1 = 45 deg

	Maneuver	
ρ, deg	Direct	Dogleg
0	1.82	1.92
30	1.92	1.86
60	2.04	1.94
90	2.47	2.14
120	2.11	2.25
150	2.05	2.18
180	2.02	2.05

maneuvers. For both cases, we have implemented covariance analyses, which we have validated by means of Monte Carlo simulations. These and Monte Carlo simulations mentioned later are all based on 100 experiments constructed with computer-generated normal distributed random numbers. Details may be found in Ref. 2.

Based on these covariance analyses, we have constructed examples with a path length $\phi = 60$ deg and $\eta_1 = 15$ and 45 deg. The results are shown in Table 2 for the worst case, that is, $\eta_1 = 45$ deg. Further, the following assumptions were made:

1) The start attitude is $\lambda_0 = 0$ deg, $\delta_0 = 10$ deg, and the assumed standard deviation of the error on the overall attitude direction is $\sigma_{\text{att}} = 0.\bar{3}$ deg $= [\sigma_\delta^2 + (\sigma_\lambda \cos\delta)^2]^{1/2}$, where the contribution of the error in attitude declination σ_δ is chosen to be equal to the perpendicular error on the latitude circle, namely, $\sigma_\lambda \cos\delta$, on a unit sphere centered around the origin of the sun coordinate system.

2) An a priori calibration error of the rhumb angle is characterized by the standard deviation $\sigma_\rho = 1$ deg.

3) An a priori calibration error of the thrust level is characterized by the standard deviation $\sigma_k = 0.03$, and consequently, for the path length, $\sigma_\phi = \phi\sigma_k$ applies.

Note from Table 2 that off-target errors of direct maneuvers and dogleg maneuvers are in fact comparable, though one would have intuitively expected an increase for doglegs whose total path lengths have in the mean increased by more than 30% according to Table 1. Thus, the dogleg slew strategy does not involve a supplementary risk in achieving the ultimate target with reasonable accuracy.

Accuracy of the Blind Estimate of the End Attitude

Even if we have two slews of different length separated in time, which enables one to compute the values of k and ϵ by Eqs. (5) and (6), we can, after the event, recompute the slews to obtain the corrected end attitude without estimating it in an other way. We call this a blind estimate of the attitude because only the initial attitude and four solar aspect angles are required and no other measurements are needed to achieve the estimation.

We have investigated the accuracy capability of dogleg maneuvers by means of a covariance analysis. First, this implies the covariance computation of the updated calibration parameters k and ϵ based on the standard deviation of the solar aspect angle measurement, which we have set to $\sigma_\theta = 0.05$ deg. Subsequently, we compute the expected attitude covariance corresponding to the attitude propagation over the corrected theoretical slews. Table 3 is compiled employing the same assumptions as in the analysis of off-target errors described before. It shows the ratio of the initial

Table 3 Ratio of the accuracy of the initial attitude knowledge over the blind attitude accuracy

ρ, deg	η_1 = 15 deg	η_1 = 25 deg	η_1 = 35 deg	η_1 = 45 deg
0	0.979	1.145	1.205	1.229
30	0.668	0.919	1.067	1.159
60	0.604	0.837	0.965	1.042
90	0.595	0.784	0.876	0.923
120	0.580	0.759	0.846	0.885
150	0.617	0.809	0.898	0.937
180	0.661	0.857	0.948	0.989

attitude ($\sigma_{\text{att}} = 0.\bar{3}$ deg) standard deviation over the standard deviation applicable to the end attitude. Values larger than 1 in Table 3 correspond to expected blind end attitude accuracies being better than the attitude knowledge accuracy before the slews. The ratios shown in Table 3 are all comprised between 0.58 and 1.23 indicating that the blind estimate has an expected accuracy that remains in the same order of magnitude as the accuracy of the initial attitude. The dogleg strategy provides a technique that allows one to slew into an orientation where attitude reconstitution conditions by conventional means must not immediately be feasible. The blind attitude is sufficiently accurate to plan for an urgent corrective slew, should the maneuver end attitude be in a contingent orientation due to a steady misperformance of the thruster(s) used. The next section gives a realistic sample of what can happen in true operations in particular with catalytic hydrazine decomposition thrusters.

Operational Experience

We consider three spin-stabilized European satellites, which have been launched successfully into a geostationary transfer orbit by an Ariane 4 rocket, namely, Meteosat Operational Program 2 (MOP2) in March 1991, MOP3 in November 1992, and Meteosat Transitional Program (MTP) in September 1997. They have the same reaction control system based on catalytic hydrazine decomposition thrusters. Also mass properties are similar. The transfer orbit operations have been conducted by the ESOC in Darmstadt, Germany. In all three cases, the spin axis had to be slewed over more than 90 deg from an attitude laying a few degrees out of the orbit plane to a direction perpendicular to the orbit plane shortly after the solid apogee engine firing and the apogee motor case ejection. Because the S-band antenna of the spacecraft is not omnidirectional, the spin axis erection slew just mentioned should not be delayed to avoid a communication blackout at a time when the spacecraft is not yet in a stable overall configuration. Moreover, the limited measurement coverage of the infrared Earth pencil beam sensors, required to estimate changing attitudes in real-time, allows limited or no attitude determination during part of the slew. If the maneuver does not perform nominally, errors in the range of expectation may move the spin axis into a final orientation without immediate infrared attitude sensor coverage, which, combined with an antenna coverage problem, may lead to an unnecessary contingency situation. The dogleg technique has been developed to overcome this problem, and it has been applied in the three cases.

The dogleg maneuver parameters, the resulting calibration figures, and the difference Δ_{att} between near real-time attitude estimation based on optical sensors and the attitude obtained by the dogleg procedure are given in Table 4. The selection of η_1 is solely based on the criterion to keep station contact during the two consecutive slews under the assumption of a nominal performance.

If the value of Δ_{att} is smaller than 1 deg we may assume that the dogleg procedure led to a correct result. Values close to 1 for k and small for ϵ indicate that the maneuver went smoothly and more or less according to predictions. This is the case for MOP2.

For MOP3, the dogleg procedure uncovered a large underperformance of the thruster (called A2) used for the spin axis erection. The whole dogleg also ended without infrared sensor coverage, but the attitude obtained revealed to be as good as the conventional attitude estimation results available later on. The poor performance of the A2 thruster was acknowledged, and one decided to avoid its usage in further operations.

For MTP, the procedure did not work satisfactorily. Fortunately, infrared sensor coverage was available throughout both slew legs.

Table 4 Comparison of three dogleg maneuvers implemented at the occasion of a larger-than-90-deg slew

Parameter	MOP2	MOP3	MTP
η_1 deg	+30.0	−15.0	−15.0
ρ_1 deg	−52.00	13.86	8.63
ϕ_1 deg	55.73	51.54	50.54
ρ_2 deg	9.09	−14.30	−22.77
ϕ_2 deg	56.40	51.27	50.64
k	1.034	0.789	1.081
ϵ deg	−1.36	+12.32	−1.98
Δ_{att}	0.40	0.58	16.02

Thereby, it could be established that the thruster performance was markedly different in each of the two legs. In the first leg, one found a 7% underperformance and an $\epsilon < 1$ deg, whereas in the second leg, one observed only a 2% underperformance, but $\epsilon \approx 5$ deg. Giving technical reasons for this unexpected behavior is beyond the scope of this paper. We can, however, identify the operational differences that prevailed at the beginning of the different spin axis erection maneuvers. The tank pressure was comparable in all three cases. It was down from the initial 23.5 ± 0.5 bars at launch to 19.6 ± 0.2 bars due to an almost equal and large spin up performed immediately after launch. If we look at the operations history of the A2 thrusters, differences are conspicuous. This thruster, with a 10-N thrust level at the beginning of life, is used for contingency sun angle corrections and all attitude slews required to achieve the apogee motor firing attitude preceding the spin axis erection. Just before spin axis erection, the satellites were spinning at approximately 85 rpm, and during a pulsed attitude maneuver, the thruster valve was opened for 97 ms each revolution. The number of little slews necessary using the A2 thruster before the spin axis erection varies from transfer orbit to transfer orbit. For MOP2, we needed three pulsed maneuvers, totaling 783 pulses. For MOP3, we required three maneuvers totaling still 233 pulses, whereas for MTP, we are speaking of two slews, one of 70 pulses and another of 14 pulses. It appears that any run in of the thrusters in space (whatever this implies technically) was complete for MOP2 and MOP3, but not for MTP. This intuitive viewpoint was confirmed in practice inasmuch as the next slew of MTP executed immediately after the dogleg by the A2 thruster performed almost the same way as in the second leg. Anyway, the dogleg was not the reason that the target was missed. A direct maneuver would have missed it equally. What we lost was the benefits of the procedure. What we learned was that confidence and repeatability of the usage of a particular thruster has to be established before one can accept reliably the idea to apply a dogleg blindly in operations.

Conclusions

A simple calibration technique for attitude slews by reaction control thrusters on spinning satellites has been presented. If one forces the application of the procedure by accepting a dogleg instead of a direct path, one will need fuel for an extra path length in the order of 5–35%. The potential error of the achieved target remains more or less unaffected by selecting a dogleg instead of a direct slew. If one can ensure that the thruster behavior is almost equal in both legs, the attitude at the end of the dogleg can be computed from the dogleg data and the initial attitude only. The quality of this estimate is good. Looking at the operational experience, the procedure worked very well if thruster performance was repeatable, but the application to MTP showed that repeatable performance is not straightforward for the first few maneuvers of a catalytic hydrazine decomposition thruster in a given mission. Thus, care is required in early operations and in interplanetary missions where the reliable performance of a dogleg may be mission critical.

Acknowledgment

The present paper is a shortened version, yet including operational experience, of a presentation made at the Flight Dynamics Symposium held in Toulouse in November 1989.

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F. H. Lutze Jr.
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